A Unified Framework for Kinematic-Dynamic Equivalence

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Abstract

Kinematic-Dynamic Equivalence is a simple phenomenological framework that describes physical systems across quantum, classical, relativistic, and cosmological scales using the dimensionless ratio $\kappa = \frac{xp}{tE} = \frac{v^2}{c^2}$, where (x) is a characteristic length, (t) is a characteristic time, (p) is momentum, (E) is energy, and (v) is the system's characteristic velocity. By scaling quantities relative to Planck units with a factor α, the framework unifies kinematic ($c_a = x/t$) and dynamic ($c_b = E/p$) scales, yielding $\kappa = 1$ for massless particles and $\kappa < 1$ for massive objects. A systematic methodology for selecting characteristic quantities ensures broad applicability. The framework describes systems like supermassive black holes, orbiting planets, relativistic electrons, quantum pair production, and inflationary cosmology, incorporating relativistic effects, spacetime curvature, quantum properties, and all fundamental forces.

Contents

1 Introduction

Unifying the description of physical systems across quantum mechanics, general relativity, and cosmology remains a central challenge in physics, particularly at Planck scales ($l_p \approx 1.616 \times 10^{-35}\text{m}$, $t_p \approx 5.391 \times 10^{-44}\text{s}$). I propose a phenomenological framework based on the dimensionless ratio:

$$\kappa = \frac{c_a}{c_b} = \frac{x/t}{E/p} = \frac{xp}{tE} = \frac{v^2}{c^2}$$

where x/t is the kinematic speed, E/p is the dynamic speed, and (v) is the system's characteristic velocity (v = c for massless particles, v < c for massive objects). Scaling physical quantities relative to Planck units—derived from the speed of light ((c)), reduced Planck constant ($\hbar$), and gravitational constant ((G)); using a factor α, this framework provides a unified description of diverse systems. It captures relativistic effects (time dilation, length contraction), spacetime curvature (black holes, early universe), quantum properties (wave-particle duality, uncertainty), and all fundamental forces relative to the Planck force ($F_p \approx 1.21 \times 10^{44}\text{N}$). This paper focuses on the framework's descriptive power, demonstrating its ability to characterize systems across physical regimes.

2 Framework Description

The framework uses the ratio $\kappa = \frac{xp}{tE} = \frac{v^2}{c^2}$ to describe physical systems by relating kinematic and dynamic scales. Below, we outline its components and methodology.

2.1 Definition of the Ratio

The ratio is defined as:

$$\kappa = \frac{xp}{tE}$$

where:

- (x): Characteristic length (e.g., radius, wavelength, units: m).
- (t): Characteristic time (e.g., period, coherence time, units: s).
- (E): Energy (e.g., relativistic energy, units: kg·m^{2}·s^{-2}).
- (p): Momentum (e.g., relativistic momentum, units: kg·m·s^{-1}).
- $c_a = x/t$: Kinematic speed (units: m·s^{-1}).
- $c_b = E/p$: Dynamic speed (units: m·s^{-1}).

The ratio is dimensionless:

$$\left[\frac{xp}{tE}\right] = \frac{(\mathrm{m}) \cdot (\mathrm{kg} \cdot \mathrm{m} \cdot \mathrm{s}^{-1})}{(\mathrm{s}) \cdot (\mathrm{kg} \cdot \mathrm{m}^2 \cdot \mathrm{s}^{-2})} = 1$$

For massless particles (v = c), $\kappa = 1$; for massive particles (v < c), $\kappa < 1$, reflecting relativistic constraints.

2.2 Selection of Characteristic Quantities

To ensure consistent application, we define a methodology for choosing (x) and (t):

- Relativistic Systems: (x) is the spatial scale (e.g., orbital radius, Schwarzschild radius), (t) is the dynamical time (e.g., orbital period x/v, light-travel time x/c). For non-uniform motion, use effective velocity from geodesic equations.

- Quantum Systems: (x) is the de Broglie wavelength ($\lambda = h/p$) or Compton wavelength ($\lambda_C = h/mc$), $t \approx \lambda/\langle v \rangle$, where $\langle v \rangle = \langle p \rangle/m$. For wave packets, $x \approx \sigma_x$ (position uncertainty),
- $t \approx \sigma_x/\langle v \rangle$.
- Cosmological Systems: $x = a(t)x_{\text{com}}$, where (a(t)) is the scale factor and $x_{\text{com}}$is comoving distance; (t) is cosmic time or Hubble time (H^{-1}).
- Complex Systems: Use expectation values or effective parameters (e.g., density matrix averages for multi-particle systems).

This methodology ensures κ is computed robustly across systems.

2.3 Planck Units and Scaling

Physical quantities are scaled relative to Planck units:

- Length: $l_p = \sqrt{\frac{\hbar G}{c^3}} \approx 1.616 \times 10^{-35}\text{m}$
- Time: $t_p = \sqrt{\frac{\hbar G}{c^5}} \approx 5.391 \times 10^{-44}\text{s}$
- Mass: $m_p = \sqrt{\frac{\hbar c}{G}} \approx 2.176 \times 10^{-8}\text{kg}$
- Energy: $E_p = \sqrt{\frac{\hbar c^5}{G}} \approx 1.956 \times 10^{9}\text{J}$
- Momentum: $p_p = \sqrt{\frac{\hbar c^3}{G}} \approx 6.524 \times 10^{27}\text{kg} \cdot \text{m/s}$
- Force: $F_p = \frac{c^4}{G} \approx 1.21 \times 10^{44}\text{N}$

Scaling:

- Mass: $m = \frac{m_p}{\alpha_m}$, $\alpha_m = \frac{m_p}{m}$
- Length: $x = \alpha_s l_p$
- Time: $t = \alpha_t t_p$

- Energy: $E = \frac{E_p}{\alpha_E}$
- Momentum: $p = \frac{p_p}{\alpha_p}$

The ratio becomes:

$$\kappa = \frac{\alpha_s \alpha_E}{\alpha_t \alpha_p}, l p_p = t_p E_p = \hbar$$

Scaling rules:

- Relativistic: $\alpha_s/\alpha_t = v/c$, $\alpha_E = \frac{E_p}{\gamma m c^2}$, $\alpha_p = \frac{p_p}{\gamma m v}$
- Quantum: $\alpha_s \approx \lambda/l_p$, $\alpha_t \approx (\lambda/\langle v \rangle)/t_p$, with $E = \sqrt{p^2c^2 + m^2c^4}$
- Cosmological: $\alpha_s = a(t)x_{\text{com}}/l_p$, $\alpha_t = t/t_p$

2.4 Descriptive Applications

The framework describes systems by computing κ:

- Massless Particles: $E = pc$, $x/t = c$, so $\kappa = \frac{c}{c} = 1$
- Massive Particles: $E = \gamma m c^2$, $p = \gamma m v$, $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$, so $\kappa = \frac{x/t}{E/p} = \frac{v}{c^2/v} = \frac{v^2}{c^2}$

Fundamental Forces:

- Gravitational: $F_{\text{grav}} = F_p \cdot \frac{\alpha_{A,m}^{-1}\alpha_{B,m}^{-1}}{\alpha_r^2}$, $\alpha_r = r/l_p$
- Electromagnetic: $F_{\text{EM}} = \alpha_{\text{EM}} F_p \cdot \frac{\beta_A \beta_B}{\alpha_r^2}$, $\alpha_{\text{EM}} \approx 1/137$
- Strong: $F_{\text{strong}} = \alpha_s F_{\text{s},0} \cdot \frac{\beta_A \beta_B}{\alpha_r^2} e^{-\frac{\alpha_r l_p}{r_0}}$, $r_0 \approx 1.4 \times 10^{-15}\text{m}$
- Weak: $F_{\text{weak}} = \alpha_w F_{\text{w},0} \cdot \frac{\beta_A \beta_B}{\alpha_r^2} e^{-\frac{\alpha_r l_p}{r_0}}$, $r_0 \approx 2.5 \times 10^{-18}\text{m}$

- Wave Properties: Wave number $k = p/\hbar = 1/(\alpha_p l_p)$, frequency $\nu = E/h = 1/(\alpha_E t_p)$
- Relativistic and Gravitational Effects: Incorporates time dilation ($t' = \gamma t_0$), length contraction ($x' = x_0/\gamma$), and curvature (Schwarzschild, FLRW metrics).

3 Descriptive Applications

The framework's descriptive power is illustrated through its application to diverse systems, computing κ and related quantities.

3.1 Astrophysical Systems

Supermassive Black Hole ($m = 8 \times 10^{36}\text{kg}$):

- $x \approx r_s = \frac{2Gm}{c^2} \approx 1.19 \times 10^{10}\text{m}$, $t \approx r_s/c \approx 3.97 \times 10^{-2}\text{s}$
- $\alpha_m \approx 2.72 \times 10^{-45}$, $\alpha_s \approx \alpha_t \approx 7.368 \times 10^{44}$
- For Hawking radiation ($v \approx c$), $\kappa \approx 1$

Earth-Sized Planet ($m = 5.972 \times 10^{24}\text{kg}$)

- Orbital radius $r = 1.5 \times 10^{11}\text{m}$, $v = \sqrt{\frac{Gm_A}{r}} \approx 5.97 \times 10^7\text{m/s} \approx 0.199c$
- $x \approx r$, $t \approx r/v \approx 2.51 \times 10^3\text{s}$
- $\alpha_s \approx 9.28 \times 10^{45}$, $\alpha_t \approx 4.66 \times 10^{46}$, $\kappa \approx (0.199)^2 \approx 0.0396$

3.2 Quantum Systems

Relativistic Electron ($m_e \approx 9.11 \times 10^{-31}\text{kg}$, $v \approx 0.99c$):

- $p \approx 1.92 \times 10^{-21}\text{kg} \cdot \text{m/s}$, $x \approx \lambda = h/p \approx 3.45 \times 10^{-13}\text{m}$
- $t \approx \lambda/v \approx 1.16 \times 10^{-21}\text{s}$
- $\alpha_s \approx 2.14 \times 10^{22}$, $\alpha_t \approx 2.15 \times 10^{22}$, $\kappa \approx (0.99)^2 \approx 0.9801$

Pair Production:

- $x \approx \lambda_C \approx 3.86 \times 10^{-13}\text{m}$, $t \approx \lambda_C/c$
- $\kappa \approx 1$, reflecting ultra-relativistic dynamics.

3.3 Cosmological Systems

Early Universe:

- $x \approx l_p$, $t \approx t_p$, $\kappa \approx 1$

Inflationary Cosmology:

- Hubble parameter $H \approx 10^{35}\text{s}^{-1}$, $x \approx t \approx H^{-1}$
- $\kappa \approx 1$, consistent with ultra-relativistic expansion.

Dark Energy Era:

- Non-relativistic matter, $\kappa < 1$, reflecting slower dynamics.

4 Discussion

4.1 Descriptive Strengths

The framework's descriptive power lies in its ability to:

- Unify Scales: Applies $\kappa = \frac{v^2}{c^2}$ across quantum ($\kappa \approx 0.9801$ for electrons), astrophysical ($\kappa \approx 0.0396$ for planets, $\kappa \approx 1$ for black holes), and cosmological ($\kappa \approx 1$ for early universe) systems.
- Incorporate Fundamental Forces: Quantifies gravitational, electromagnetic, strong, and weak forces in a consistent Planck-scaled formalism.
- Handle Complexity: Adapts to complex systems via expectation values or effective parameters, ensuring broad applicability.
- Capture Physical Effects: Describes relativistic effects (time dilation, length contraction), spacetime curvature, and quantum properties (wave-particle duality).

Its simplicity, using a single dimensionless ratio, makes it accessible and computationally straightforward, requiring only characteristic quantities and Planck-unit scaling.

4.2 Contributions

- Unified Description: Provides a common language for interdisciplinary research, bridging quantum mechanics, general relativity, and cosmology.
- Phenomenological Constraint: Offers a framework to characterize systems and guide experimental analysis, particularly at extreme scales.
- Versatility: Applies to diverse systems, from particles to black holes to the universe's evolution.

4.3 Limitations

- Descriptive Nature: Reproduces system properties but does not derive fundamental laws (e.g., Einstein's equations, quantum field theory).
- Dependence on Characteristic Quantities: Requires careful selection of (x, t, p, E), though the methodology mitigates ambiguity.
- Limited Formalism: Lacks a complete theoretical structure, functioning as a phenomenological tool rather than a predictive theory.

4.4 Future Directions

- Refining Applications: Extend to additional systems (e.g., neutron stars, quantum fluids).
- Experimental Guidance: Use κ to analyze high-energy experiments or astrophysical observations.
- Interdisciplinary Integration: Incorporate into existing models (e.g., loop quantum gravity, string theory) as a descriptive constraint.

5 Conclusion

The ratio $\kappa = \frac{xp}{tE} = \frac{v^2}{c^2}$ provides a robust phenomenological framework for describing physical systems across quantum, relativistic, and cosmological regimes. By scaling characteristic quantities relative to Planck units, it unifies kinematic and dynamic scales, capturing relativistic, quantum, and gravitational effects.