A Unified Framework for Kinematic-Dynamic Equivalence

Gabriel Stuckey

13 Jun 2025 (updated: 6, 16 Jul 2025)

Abstract

Kinematic-Dynamic Equivalence is a phenomenological framework that describes physical systems across quantum, classical, relativistic, and cosmological scales using the dimensionless ratio $\kappa = \frac{xp}{tE} = \frac{v^2}{c^2}$, where (x) is a characteristic length, (t) is a characteristic time, (p) is momentum, (E) is energy, and (v) is the system's characteristic velocity. By scaling quantities relative to Planck units with a factor α, the framework unifies kinematic ($c_a = x/t$) and dynamic ($c_b = E/p$) scales, yielding $\kappa = 1$ for massless particles and $\kappa < 1$ for massive objects. A systematic methodology for selecting characteristic quantities ensures broad applicability. The framework describes systems like supermassive black holes, orbiting planets, relativistic electrons, quantum pair production, and inflationary cosmology, incorporating relativistic effects, spacetime curvature, quantum properties, and all fundamental forces. Its simplicity, stochastic modeling, electromagnetic interactions, and cross-regime mappings, supported by heuristic approximations where noted, enhance its descriptive power.

Contents

1 Introduction

Unifying the description of physical systems across quantum mechanics, general relativity, and cosmology remains a central challenge in physics, particularly at Planck scales ($l_p \approx 1.616 \times 10^{-35}$m, $t_p \approx 5.391 \times 10^{-44}$s). I propose a phenomenological framework based on the dimensionless ratio:

$$\kappa = \frac{c_a}{c_b} = \frac{x/t}{E/p} = \frac{xp}{tE} = \frac{v^2}{c^2}$$

where x/t is the kinematic speed, E/p is the dynamic speed, and (v) is the system's characteristic velocity ($v = c$ for massless particles, $v < c$ for massive objects). Scaling physical quantities relative to Planck units, derived from the speed of light ((c)), reduced Planck constant ($\hbar$), and gravitational constant ((G)), using a factor α, this framework provides a unified description of diverse systems. It captures relativistic effects (time dilation, length contraction), spacetime curvature (black holes, early universe), quantum properties (wave-particle duality, uncertainty), and all fundamental forces relative to the Planck force ($F_p \approx 1.21 \times 10^{44}$N). Heuristic approximations, such as pseudo-charge scaling for photons and expectation values for stochastic processes, are used where noted to maintain simplicity. This paper focuses on the framework's descriptive power, emphasizing simplicity, stochastic modeling, electromagnetic interactions, and cross-regime unification.

2 Framework Description

The framework uses the ratio $\kappa = \frac{xp}{tE} = \frac{v^2}{c^2}$ to describe physical systems by relating kinematic and dynamic scales. Below, we outline its components and methodology.

2.1 Definition of the Ratio

The ratio is defined as:

$$\kappa = \frac{xp}{tE}$$

where:

- (x): Characteristic length (e.g., radius, wavelength, units: m)
- (t): Characteristic time (e.g., period, coherence time, units: s)
- (E): Energy (e.g., relativistic energy, units: kg·m^{2}·s^{-2})
- (p): Momentum (e.g., relativistic momentum, units: kg·m·s^{-1})
- $c_a = x/t$: Kinematic speed (units: m·s^{-1})
- $c_b = E/p$: Dynamic speed (units: m·s^{-1})

The ratio is dimensionless:

$$\left[\frac{xp}{tE}\right] = \frac{(\mathrm{m})\cdot(\mathrm{kg}\cdot\mathrm{m}\cdot\mathrm{s}^{-1})}{(\mathrm{s})\cdot(\mathrm{kg}\cdot\mathrm{m}^{2}\cdot\mathrm{s}^{-2})} = 1$$

For massless particles ($v = c$), $\kappa = 1$; for massive particles ($v < c$), $\kappa < 1$, reflecting relativistic constraints.

2.2 Selection of Characteristic Quantities

To ensure consistent application, we define a methodology for choosing (x) and (t):

- Relativistic Systems: (x) is the spatial scale (e.g. orbital radius, Schwarzschild radius), (t) is the dynamical time (e.g. orbital period x/v, light-travel time x/c). For non-uniform motion, use effective velocity from geodesic equations derived from the spacetime geometry in general relativity.
- Quantum Systems: (x) is the de Broglie wavelength ($\lambda = h/p$) or Compton wavelength ($\lambda_C = h/mc$), $t \approx \lambda/\langle v\rangle$, where $\langle v\rangle = \langle p\rangle/m$. For wave packets, $x \approx \sigma_x$(position uncertainty), $t \approx \sigma_x/\langle v\rangle$.
- Cosmological Systems: $x = a(t)x_{\mathrm{com}}$, where (a(t)) is the scale factor and $x_{\mathrm{com}}$is comoving distance; (t) is cosmic time or Hubble time (H^{-1}).
- Complex Systems: Use expectation values or effective parameters (e.g., density matrix averages for multi-particle systems), as a heuristic approach for stochastic processes.

This methodology ensures κ is computed robustly across systems.

2.3 Planck Units and Scaling

Physical quantities are scaled relative to Planck units:

- Length: $l_p = \sqrt{\frac{\hbar G}{c^3}} \approx 1.616 \times 10^{-35}\text{m}$
- Time: $t_p = \sqrt{\frac{\hbar G}{c^5}} \approx 5.391 \times 10^{-44}\text{s}$
- Mass: $m_p = \sqrt{\frac{\hbar c}{G}} \approx 2.176 \times 10^{-8}\text{kg}$
- Energy: $E_p = \sqrt{\frac{\hbar c^5}{G}} \approx 1.956 \times 10^{9}\text{J}$
- Momentum: $p_p = \sqrt{\frac{\hbar c^3}{G}} \approx 6.524 \times 10^{27}\text{kg} \cdot \text{m/s}$
- Force: $F_p = \frac{c^4}{G} \approx 1.21 \times 10^{44}\text{N}$

Scaling:

- Mass: $m = \frac{m_p}{\alpha_m}$, $\alpha_m = \frac{m_p}{m}$
- Length: $x = \alpha_s l_p$
- Time: $t = \alpha_t t_p$
- Energy: $E = \frac{E_p}{\alpha_E}$
- Momentum: $p = \frac{p_p}{\alpha_p}$
- Charge: $\alpha_q = \frac{q}{e}$, where $e \approx 1.602 \times 10^{-19}\text{C}$ is the electron charge

The electron charge (e) is chosen as the reference for $\alpha q = q/e$ because it is the fundamental unit of charge in quantum electrodynamics (QED), providing a standardized, universal scale for electromagnetic interactions across particles (e.g., electrons, protons). This choice simplifies scaling of the gyromagnetic ratio ($\gamma \approx \alpha_q e/m$) and force strengths ($F_{\text{EM}} \propto \alpha_q^2 \cdot \alpha_{\text{EM}}$), ensuring consistency with established models like QED (4.9).

The ratio becomes:

$$\kappa = \frac{\alpha_s \alpha_p}{\alpha_t \alpha_E}, l_p p_p = t_p E_p = \hbar$$

For massive particles, the gyromagnetic ratio ($\gamma \approx \alpha_q e/m$) influences energy shifts in stochastic processes (4.6). For photons, a heuristic pseudo-charge scaling ($\sim \nu/c$) is used as a phenomenological analogy to relate frequency to electromagnetic field effects. These scalings unify quantum, relativistic, and cosmological regimes (4.7).

2.4 Framework Applications

The framework describes systems by computing κ:

- Massless Particles: $E = pc$, $x/t = c$, so $\kappa = \frac{c}{c} = 1$
- Massive Particles: $E = \gamma mc^2$, $p = \gamma mv$, $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$, so $\kappa = \frac{x/t}{E/p} = \frac{v}{c^2/v} = \frac{v^2}{c^2}$

Fundamental Forces:

- Gravitational: $F_{\text{grav}} = F_p \cdot \frac{\alpha_{A,m}^{-1}\alpha_{B,m}^{-1}}{\alpha_r^2}$, $\alpha_r = r/l_p$
- Electromagnetic: $F_{\text{EM}} = \alpha_{\text{EM}} F_p \cdot \frac{\beta_A \beta_B}{\alpha_r^2}$, $\alpha_{\text{EM}} \approx 1/137$
- Strong: $F_{\text{strong}} = \alpha_s F_{s,0} \cdot \frac{\beta_A \beta_B}{\alpha_r^2} e^{-\frac{\alpha_r l_p}{r_0}}$, $r_0 \approx 1.4 \times 10^{-15}$m
- Weak: $F_{\text{weak}} = \alpha_w F_{w,0} \cdot \frac{\beta_A \beta_B}{\alpha_r^2} e^{-\frac{\alpha_r l_p}{r_0}}$, $r_0 \approx 2.5 \times 10^{-18}$m

Force-specific length scales, defined as $x_f \approx \lambda \cdot \alpha_f$, adjust the characteristic length by the coupling strength of each force (α_f, e.g.,$\alpha_{\text{EM}} \approx 1/137$) to reflect interaction ranges. These scales connect to κ by modifying the characteristic length $x \approx x_f$ in the ratio $\kappa = \frac{xp}{tE}$, enabling the framework to capture force-driven dynamics (4.7).

Wave Properties: Wave number $k = \frac{p}{\hbar} = \frac{1}{(\alpha_p l_p)}$, frequency $\nu = \frac{E}{h} = \frac{1}{(\alpha_E t_p)}$.

Relativistic and Gravitational Effects: Incorporates time dilation ($t' = \gamma t_0$), length contraction ($x' = x_0/\gamma$), and curvature (Schwarzschild, FLRW metrics).

3 Descriptive Applications

The kinematic-dynamic equivalence framework applies the dimensionless ratio $\kappa = \frac{xp}{tE} = \frac{v^2}{c^2}$ to describe physical systems across astrophysical, quantum, and cosmological regimes by selecting appropriate characteristic quantities (2.2). This section outlines the process of calculating κ in each regime, emphasizing the methodology for choosing length ((x)), time ((t)), momentum ((p)), and energy ((E)), and how these yield κ to reflect system dynamics. A summary table highlights typical κ values, illustrating the framework's ability to unify diverse scales.

3.1 Astrophysical Systems

In astrophysical systems, such as black holes or orbiting planets, κ is calculated by selecting characteristic quantities that reflect gravitational and orbital dynamics. For a black hole, the characteristic length is the Schwarzschild radius ($x \approx 2GM/c^2$), representing the event horizon, and the time is the light-travel time across this radius ($t \approx x/c$). Momentum and energy are derived from the system's dynamics, typically assuming relativistic velocities ($v \approx c$) for processes like Hawking radiation. The ratio $\kappa = \frac{xp}{tE}$ simplifies to $\kappa \approx v^2/c^2$, yielding $\kappa \approx 1$ for relativistic systems. For an orbiting planet, the characteristic length is the orbital radius ($x \approx r$), and the time is the orbital period divided by velocity ($t \approx r/v$), where velocity is determined by gravitational equilibrium ($v = \sqrt{GM/r}$, with (M) as the central mass). Momentum is $p \approx mv$ (or γmv for relativistic cases), and energy is approximated as $E \approx mc^2$ in the framework's relativistic context (2.1), ensuring $\kappa \approx v^2/c^2$. This typically yields $\kappa < 1$, reflecting non-relativistic orbital dynamics. The relativistic approximation is preferred to align with the framework's focus on unifying scales through v^2/c^2.

3.2 Quantum Systems

In quantum systems, such as relativistic electrons or pair production, κ is computed using quantum mechanical quantities. For an electron, the characteristic length is the de Broglie wavelength ($x \approx \lambda = h/p$), where momentum $p = \gamma mv$ accounts for relativistic effects ($\gamma = 1/\sqrt{1 - v^2/c^2}$). The time is chosen as $t \approx \lambda/v$, representing the time to traverse the wavelength at the electron's velocity, consistent with quantum mechanics (Section 2.2). Energy is $E = \gamma mc^2$, yielding $\kappa \approx v^2/c^2$, typically close to 1 for relativistic electrons. For pair production, the characteristic length is the Compton wavelength ($x \approx \lambda_C = h/(mc)$), and the time is $t \approx \lambda_C/c$, reflecting the ultra-relativistic nature of the process ($v \approx c$). Momentum and energy are derived from QED, with $\kappa \approx 1$. The framework uses expectation values for stochastic processes (e.g., scattering probabilities), ensuring κ captures quantum dynamics.

3.3 Cosmological Systems

In cosmological systems, such as the early universe or dark energy era, κ reflects large-scale dynamics. For the early universe, Planck-scale quantities are used ($x \approx l_p$, $t \approx t_p$), with $\kappa \approx 1$ due to ultra-relativistic conditions. During inflation, the characteristic length and time are set by the Hubble parameter ($x \approx t \approx H^{-1}$), yielding $\kappa \approx 1$ for rapid expansion. In the dark energy era, non-relativistic matter (e.g., galaxies) uses the galactic radius as the characteristic length ($x \approx r$) and time as $t \approx r/v$, where (v) is the peculiar velocity. Momentum is $p \approx mv$, and energy is approximated as $E \approx mc^2$ (2.1), giving $\kappa \approx v^2/c^2$, typically much less than 1 due to low velocities. The relativistic approximation ensures consistency across regimes.

4 Discussion

4.1 Descriptive Strengths

The framework's strength lies in its ability to describe a wide variety of systems using a single dimensionless ratio κ, capturing quantum, relativistic, and gravitational effects (2.2). By incorporating electromagnetic interactions through the gyromagnetic ratio and charge scaling, κ describes spin-dependent dynamics for massive particles and polarization effects for photons, using heuristic pseudo-charge scaling where noted. Cross-regime mapping standardizes quantity definitions, unifying quantum, relativistic, and cosmological systems, while maintaining simplicity. The framework tracks the evolution of κ and interval perceptions across inertial frames via force-specific cross-regime mapping and relativistic effects, leveraging heuristic expectation values for stochastic processes.

4.2 Contributions

- Unified Description: Provides a common language for interdisciplinary research, bridging quantum mechanics, general relativity, and cosmology through κ, enhanced by frequency, wavelength, and stochastic modeling, electromagnetic interactions, and cross-regime mapping
- Phenomenological Constraint: Offers a framework to characterize systems and guide experimental analysis, particularly at extreme scales
- Versatility: Applies to diverse systems, from particles to black holes to the universe's evolution, unified by force-specific and relativistic effects

4.3 Limitations

- Descriptive Nature: Reproduces system properties but does not derive fundamental laws
- Dependence on Characteristic Quantities: Requires careful selection of ((x, t, p, E)), though the methodology mitigates ambiguity (2.2)
- Limited Formalism: Lacks a complete theoretical structure, functioning as a phenomenological tool rather than a predictive theory, with heuristic approximations noted

4.4 Future Directions

Future directions include validating κ through high-energy experiments (e.g., electron scattering), astrophysical observations (e.g., black hole orbits), and cosmological studies (e.g., CMB fluctuations). Incorporating electromagnetic interactions via gyromagnetic ratio and charge scaling and formalizing polarization for photons can enhance stochastic modeling. Cross-regime mapping, including force-specific scales and relativistic effects, offers a pathway to standardize quantity definitions, fostering interdisciplinary applications while maintaining simplicity.

4.5 Simplicity as a Core Principle

The kinematic-dynamic equivalence framework's strength lies in its simplicity, using a single dimensionless ratio, $\kappa = \frac{xp}{tE} = \frac{v^2}{c^2}$, to describe physical systems across quantum, relativistic, and cosmological scales (2.2). By defining characteristic quantities ($x \approx \lambda$, $t \approx \lambda/v$) relative to Planck units (Section 2.3) and leveraging outputs from established models (e.g., QED, general relativity), the framework avoids complex dynamical equations, ensuring accessibility for interdisciplinary applications.

For example, in quantum systems like a relativistic electron ($v \approx 0.99c$), $\kappa \approx 0.9801$ is computed using $x \approx \lambda = h/p$, $t \approx \lambda/v$, $p = \gamma m_e v$, and $E = \gamma m_e c^2$, with external models providing probabilities (4.6–4.7). This simplicity underpins the framework's ability to unify diverse regimes, serving as a common language for quantum, relativistic, and cosmological systems (4.2), with further refinements proposed for standardized quantity selection (4.4).

4.6 Interactions

The framework captures stochastic and electromagnetic interactions through expectation values and specialized scaling, enhancing its descriptive power for complex systems. For stochastic processes, $\kappa = \frac{\langle x\rangle\langle p\rangle}{\langle t\rangle\langle E\rangle}$ is computed using heuristic expectation values from probabilistic distributions (e.g., QED cross-sections), relying on external models to maintain simplicity (4.5).

For example, in Mott scattering of an electron ($v \approx 0.99c$), a velocity shift to $v' \approx 0.95c$ updates $\kappa \approx 0.9801 \to 0.9025$. Electromagnetic interactions are incorporated via the gyromagnetic ratio and charge scaling. For massive particles like electrons ($m_e \approx 9.11 \times 10^{-31}$kg, spin $s = \hbar/2$), the gyromagnetic ratio $\gamma_e \approx \alpha_q e/m_e$, where $\alpha_q = q/e$ ($e \approx 1.602 \times 10^{-19}$C), scales energy shifts in a magnetic field: $\Delta E = \mu_B B \frac{\langle S_z\rangle}{\hbar}$, with $\mu_B = \frac{e\hbar}{2m_e}$, adjusting κ slightly (2.3). The electromagnetic coupling constant $\alpha_{\mathrm{EM}} \approx 1/137$ scales force strength as $F_{\mathrm{EM}} \propto \alpha_q^2 \cdot \alpha_{\mathrm{EM}}$, with $x_{\mathrm{EM}} \approx \lambda \cdot \alpha_{\mathrm{EM}}$, $t_{\mathrm{EM}} \approx x_{\mathrm{EM}}/v$ reflecting interaction ranges (4.7). For photons ($\kappa = 1$), a heuristic pseudo-charge scaling ($\sim \nu/c$) serves as a phenomenological analogy to relate frequency to field strength, justified by its ability to map interaction dynamics via $\langle\lambda\rangle = \sum_\epsilon w_\epsilon \lambda_\epsilon$, $\langle\nu\rangle = c/\langle\lambda\rangle$, where polarization weights w_ϵ adjust outcomes while preserving

$\kappa = 1$ (4.6). This approach unifies quantum and relativistic systems by integrating stochastic and electromagnetic effects into κ.

4.7 Cross-Regime Unification

The framework unifies quantum, relativistic, and cosmological regimes through cross-regime mapping, incorporating force-specific length scales and tracking κ evolution. Characteristic quantities are standardized: $x \approx \lambda = h/p$, $t \approx \lambda/v$ for quantum systems; $x \approx r$ or $r_s = 2GM/c^2$, $t \approx r/v$ for relativistic systems; and $x \approx a(t)x_{\text{com}}$, $t \approx H^{-1}$ for cosmological systems (2.2). Force-specific length scales, $x_f \approx \lambda \cdot \alpha_f$, adjust (x) by coupling strengths

- gravitational: $\alpha_g \approx (m/m_p)^2(l_p/r)^2$
- electromagnetic: $\alpha_{\text{EM}} \approx 1/137$
- strong: $\alpha_s \approx 1$
- weak: $\alpha_w \approx 10^{-6}$) relative to the Planck force (2.4), with $t_f \approx x_f/v$.

In stochastic processes, $\kappa = \frac{\langle x_f \rangle \langle p \rangle}{\langle t_f \rangle \langle E \rangle}$ tracks force-driven changes, such as electromagnetic scattering adjusting κ for electrons or gravitational effects shifting planetary κ. This mapping elucidates κ evolution and perceptual differences in time and length intervals across inertial frames. For example, electromagnetic interactions adjust $\langle v \rangle$ for electrons, while gravitational interactions dominate for planets. For photons, $\langle \lambda \rangle$ changes map dynamics while $\kappa = 1$ remains fixed (4.6). These effects are unified by scaling lengths across regimes, enabling interdisciplinary comparisons (4.2).

4.8 Relativistic Effects

The framework embeds the Lorentz factor, $\gamma = (1 - v^2/c^2)^{-1/2} = 1/\sqrt{1-\kappa}$, to incorporate relativistic effects. For massive particles, γ quantifies time dilation ($t' = \gamma t_0$) and length contraction ($x' = x_0/\gamma$), influencing $\langle p \rangle = \gamma m v$, $\langle E \rangle = \gamma m c^2$. For photons ($\kappa = 1$), $\gamma \to \infty$, consistent with their relativistic nature. In stochastic processes, $\gamma = 1/\sqrt{1 - \langle \kappa \rangle}$ adjusts expectation values (Section 4.6), clarifying interval perceptions across frames, such as between quantum and relativistic systems, while maintaining simplicity (4.5).

5 Conclusion

The kinematic-dynamic equivalence framework provides a unified, descriptive approach to physical systems across quantum, relativistic, and cosmological scales through $\kappa = \frac{xp}{tE} = \frac{v^2}{c^2}$. By incorporating frequency and wavelength for photons, heuristic expectation values for stochastic processes, gyromagnetic ratio and charge scaling with a heuristic pseudo-charge for photons, cross-regime mapping, force-specific mapping grounded in coupling constants, and the Lorentz factor $\gamma = 1/\sqrt{1-\kappa}$, the framework enhances its ability to describe spin-dependent and electromagnetic interactions. These additions maintain simplicity while unifying diverse regimes, leveraging external models to bridge interdisciplinary research. Force-specific mapping elucidates κ evolution and interval differences, embedding relativistic effects to unify stochastic process modeling across scales. The framework's emphasis on a single dimensionless ratio and cross-regime mapping may inspire new theoretical insights by providing a simplified lens to identify universal patterns in complex systems, potentially guiding the development of hybrid models that integrate quantum field theory, general relativity, and cosmology, or informing experimental designs to test scaling relationships at extreme scales (4.4).